Adjusted modified internal rate of return – Another way to calculate a money weighted rate of return

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by: Dr. Stefan J. Illmer

Illmer Investment Performance Consulting AG

Weinbergstrasse 28

CH – 8200 Schaffhausen

stefan.illmer@iipc-ag.com

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INTRODUCTION

It has been several years since Wolfgang Marty and I published our article on the decomposition of the money weighted rate of return (MWR) in the Journal of Performance Measurement.[1] In the meantime, our approach has made it into the literature on performance attribution.[2] However, little has been written about the MWR of investment portfolios in the interim, which is the reason for publishing this article.

This article is a summary of a presentation I gave in 2013 and addresses one of the main criticisms of using the internal rate of return (IRR) as a method to calculate a "true" MWR: the implicit reinvestment and financing assumptions. This criticism is justified because, on the one hand, these specific assumptions do not make economic sense and, on the other hand, this may lead to more than one solution or even no solution for the IRR. But there are recognized solutions for this – like the visualization of financial implications method VOFI – which are used in investment planning, budgeting, or appraisal. These corresponding methods use explicit reinvestment and financing assumptions, which means that the mathematical problem is reduced to two points and the return can be determined unambiguously. However, there are different possibilities in which the explicit reinvestment and financing assumptions can be considered within the calculation of the rate of return. Considering the concept for calculating an IRR, in principle there are three different possibilities to treat cash flows:

- Discounting all interim cash flows to the beginning of the measurement period.
- Compounding all interim cash flows to the end of the measurement period.
- Discounting all interim cash inflows to the beginning of the measurement period and compounding all interim cash outflows to the end of the measurement period.

In case of the IRR the treatment of cash flows does not have an impact on the final result as cash flows are discounted or compounded at the same rate. If using explicit reinvestment and financing assumptions the actual return is dependent on the treatment of the interim cash flows. This article argues that there is another specific return measurement methodology, the adjusted modified internal rate of return (AMIRR), that simulates or considers relevant economic conditions, compounds all cash flows to the end of the measurement period and is thus suitable for calculating historical MWRs.

INTERNAL RATE OF RETURN

The internal rate of return (IRR) is an MWR and is called a "true" MWR as it is a precise method for calculating an MWR that solves the full calculation problem and is no approximation of the MWR. For a given measurement period, the IRR is the average rate of return that causes the ending market value and the interim external cash flows to be discounted to the beginning market value. Due to its specific compounding characteristics, the IRR is also the average rate of return that causes the beginning market value and interim external cash flows to grow to the ending market value. With this in mind, the IRR is the rate of return on the

initial investment that was earned assuming that cash flows are financed or reinvested using the following assumptions:[3]

- Cash inflows are financed at an interest rate (finance rate) that is identical to the IRR.
- Cash outflows are reinvested at an interest rate (reinvestment rate) that is identical to the IRR.

When discounting the ending market value and the interim cash flows to equal the beginning market value, the formula for deriving the annualized IRR is as follows (**formula 1**):

$$\mathrm{BMV}_0 = \frac{\mathrm{EMV}_\mathrm{T}}{(1+\mathrm{IRR})^{\mathrm{Y_T}}} - \sum \frac{\mathrm{CI}_\mathrm{t}}{(1+\mathrm{IRR})^{\mathrm{Y_{t-0}}}} + \sum \frac{\mathrm{CO}_\mathrm{t}}{(1+\mathrm{IRR})^{\mathrm{Y_{t-0}}}}$$

with

IRR = Internal rate of return (annualized) for a single period 0 to T,

BMV_0 = Market value at the beginning of the period,

EMV_T = Market value at the end of the period,

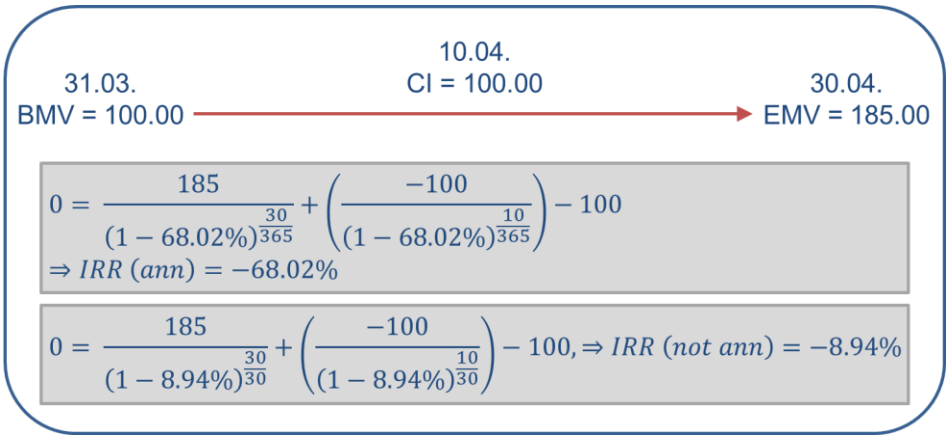
CI_t = Cash inflow at date t,

CO_t = Cash outflow at date t,

$\mathrm{Y_T}$ = Length of measurement period (measured in years – 365 days),

$\mathrm{Y_{t-0}}$ = Length of period between the beginning of the measurement period and the date of the cash flow (measured in years – 365 days).

EXAMPLE 1 – IRR with discounting cash flows



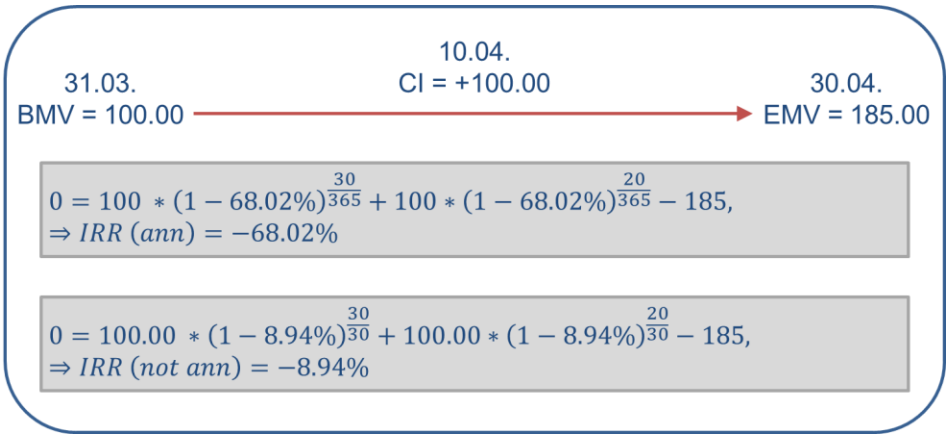
When compounding the beginning market value and the interim cash flows to equal the ending market value, the formula for deriving the annualized IRR is as follows (**formula 2**):

$$\mathrm{EMV}_\mathrm{T} = \mathrm{BMV}_0 \times (1+\mathrm{IRR})^{\mathrm{Y_T}} + \sum \mathrm{CI}_\mathrm{t} \times (1+\mathrm{IRR})^{\mathrm{Y_{T-t}}} - \sum \mathrm{CO}_\mathrm{t} \times (1+\mathrm{IRR})^{\mathrm{Y_{T-t}}}$$

with

$\mathrm{Y_{T-t}}$ = Length of period between the date of the cash flow and the end of the measurement period (measured in years – 365 days).

EXAMPLE 2 – IRR with compounding cash flows



It should be noted here that both variants lead to the same result for the IRR.

Furthermore a few critical remarks about the IRR:

- Considering the financing costs of the cash inflows and the reinvestment income of the cash outflows, for the IRR the invested capital equals the initial investment.
- IRR is an average rate of return based on actual cash flows rather than actual interim values.
- Implied interim profit and loss is not the same as the profit and loss reported on the balance sheet.
- IRR implies unrealistic reinvestment and financing assumptions as the IRR is a result of the amount and timing of cash flows.
- IRR cannot always be calculated and is sometimes ambiguous as there maybe multiple solutions or even no solution.

MODIFIED INTERNAL RATE OF RETURN

The modified internal rate of return (MIRR) is a modification of the IRR as it uses explicit reinvestment and financing assumptions. It is therefore an MWR and also called a "true" MWR because it is a precise method for calculating an MWR that solves the full calculation problem and is no approximation of the MWR.[4]

For a given measurement period, the MIRR is the discount rate that makes the investments (cash inflows) equal to the future market value of the cash flows from the investment (cash outflows). In other words, the MIRR is the return that causes the sum of the beginning market value and the discounted interim cash inflows to grow to the sum of the compounded interim cash outflows and the ending market value. With this in mind, the MIRR is the rate of return that was earned assuming that cash flows are financed or reinvested using the following explicit assumptions:

- Cash inflows are financed at a specific average interest rate (finance rate) that is constant over time and does not have to be identical to the IRR.
- Cash outflows are reinvested at a specific average interest rate (reinvestment rate) that is constant over time and does not have to be identical to the IRR.

By using explicit reinvestment assumptions, the MIRR resolves the two main issues with the IRR methodology:

- The implicit reinvestment and financing assumptions.
- The problem of having multiple solutions or no solution.

The formula for calculating the annualized MIRR is as follows (**formula 3**):

$$\text{MIRR} = \left(\frac{\begin{matrix} Ending\ market\ value + \\ future\ value\ of\ all\ cash\ outflows \end{matrix}}{\begin{matrix} Beginning\ market\ value + \\ present\ value\ of\ all\ cash\ inflows \end{matrix}} \right)^{\left(\frac{1}{Y_T}\right)} - 1$$

or

$$\text{MIRR} = \left(\frac{\text{EMV}_T + \sum \text{CO}_t * \left(1 + \text{RIR}_{0,T}\right)^{Y_{T-t}}}{\text{BMV}_0 + \sum \frac{\text{CI}_t}{\left(1 + \text{FR}_{0,T}\right)^{Y_{t-0}}}} \right)^{\left(\frac{1}{Y_T}\right)} - 1$$

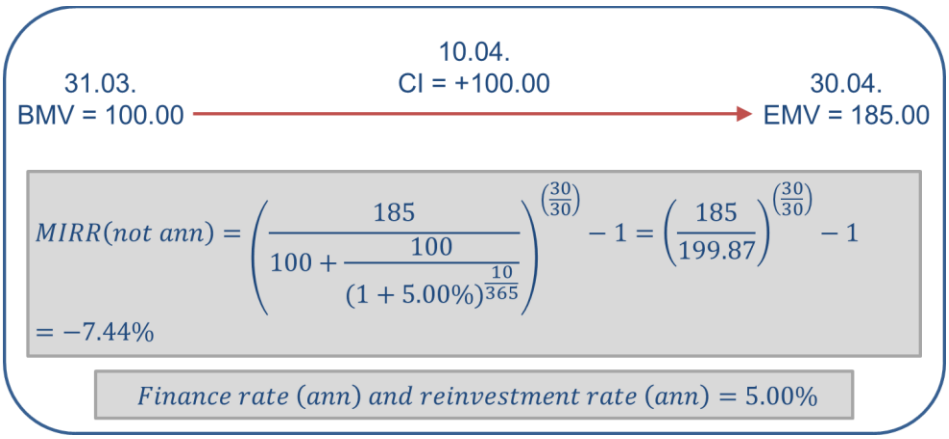
with

MIRR = Modified internal rate of return (annualized) for a single period 0 to T,

$\text{RIR}_{0,T}$ = Reinvestment rate (annualized) for the period 0 to T,

$\text{FR}_{0,T}$ = Finance rate (annualized) for the period 0 to T.

EXAMPLE 3 – MIRR with cash inflow



In example 3, the MIRR is not annualized, and Y_T is 30 days as the time period is less than a year.

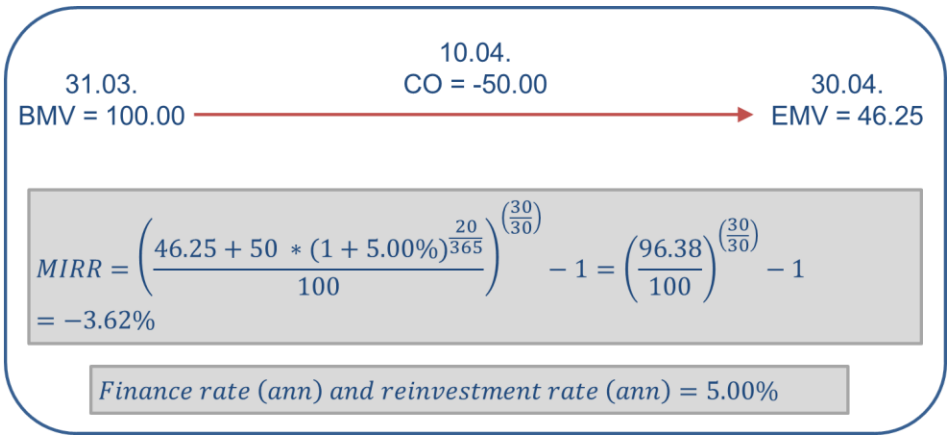
Furthermore a few critical remarks about the MIRR:

- Considering the financing costs of the cash inflows and the reinvestment income of the cash outflows, for the MIRR the invested capital does not equal the initial investment but the sum of the beginning market value and the discounted cash inflows or in other words the present value of the total invested capital.
- MIRR is an average return based on realistic reinvestment and financing assumptions.
- MIRR is an average return based on actual cash flows rather than actual interim values.
- MIRR can always be calculated and is unique – means has no problem with multiple solutions.
- MIRR depends on the rates used for the finance rate and the reinvestment rate.
- MIRR can be calculated using various reinvestment and financing assumptions – e.g., risk-free interest rate, benchmark return, target return, etc.
- MIRR uses the unrealistic assumption that the finance rate and the reinvestment rate are constant over the entire measurement period.

- MIRR discounts all cash inflows to the beginning and all cash outflows to the end of the measurement period which implicitly influences the invested capital and with this the actual return. The invested capital ignores the financed or borrowed capital and equals the gross or leveraged capital. This is inconsistent with the compounded cash outflows which are not ignored but instead increasing the invested capital at the end of the measurement period.

The last remark can best be understood by considering the case with one cash inflow illustrated by example 3 and the case with one cash outflow illustrated by example 4. The cash inflow increases the invested capital in example 3 by 99.87 (ignoring the amount borrowed of negative 100.00) and the cash outflow increases the market value at the end of the measurement period in example 4 by 50.13 (ignoring that the amount reinvested of positive 50.00 left the portfolio). To make the case consistent, the funding of the cash inflow should also be considered as negative capital in the amount of 100.00 – but this adjustment would change the capital invested and therefore as well the actual return.

EXAMPLE 4 – MIRR with cash outflow



ADJUSTED MODIFIED INTERNAL RATE OF RETURN

The adjusted modified internal rate of return (AMIRR) addresses the last critical remark to the MIRR mentioned and explained above. The AMIRR is a modification of the IRR as it uses explicit reinvestment and financing assumptions. It is therefore an MWR and also called a "true" MWR because it is a precise method for calculating an MWR that solves the full calculation problem and is no approximation of the MWR.

For a given measurement period, the AMIRR is the return that causes the beginning market value to grow to the sum of the compounded interim cash flows and the ending market value. With this in mind, the AMIRR is the rate of return on the initial investment that was earned assuming that cash flows are financed or reinvested using the following assumptions:

- Cash inflows are financed at a specific interest rate (finance rate) that may vary over time and does not have to be identical to the IRR.
- Cash outflows are reinvested at a specific interest rate (reinvestment rate) that may vary over time and does not have to be identical to the IRR.

By using explicit reinvestment assumptions, the AMIRR resolves the two main issues with the IRR methodology:

- The implicit reinvestment and financing assumptions.
- The problem of having multiple solutions or no solution.

The formula for calculating the annualized AMIRR is as follows (**formula 4**):

$$\text{AMIRR} = \left(\frac{\begin{array}{c} Ending\ market\ value + \\ future\ value\ of\ all\ cash\ outflows - \\ future\ value\ of\ all\ cash\ inflows \end{array}}{Beginning\ market\ value} \right)^{\left(\frac{1}{Y_T}\right)} - 1$$

or

$$\text{AMIRR} = \left(\frac{\begin{array}{c} \text{EMV}_T + \sum \text{CO}_t * \left(1 + \text{RIR}_{t,T}\right)^{Y_{T-t}} - \\ \sum \text{CI}_t * \left(1 + \text{FR}_{t,T}\right)^{Y_{T-t}} \end{array}}{\text{BMV}_0} \right)^{\left(\frac{1}{Y_T}\right)} - 1$$

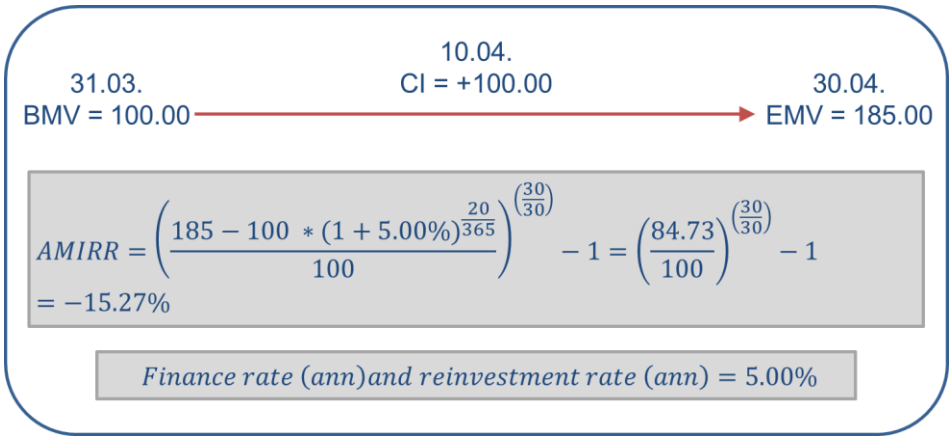
with

AMIRR = Adjusted modified internal rate of return (annualized) for a single period 0 to T,

$\text{RIR}_{t,T}$ = Reinvestment rate (annualized) for the period t to T,

$\text{FR}_{t,T}$ = Finance rate (annualized) for the time period t to T.

EXAMPLE 5 – AMIRR with cash inflow



In example 5, the AMIRR is not annualized, and Y_T is 30 days as the time period is less than a year.

Furthermore a few critical remarks about the AMIRR:

- AMIRR is an average rate of return based on realistic reinvestment and financing assumptions.
- AMIRR is an average rate of return not based on the actual interim values but on the actual cash flows.
- AMIRR can always be calculated and is unique – means has no issue with multiple solutions.
- AMIRR depends on the rates used for the finance rate and the reinvestment rate.
- AMIRR can be calculated using different reinvestment assumptions – e.g., risk free rate, benchmark return, target return, etc. – where in addition the rates may vary throughout the measurement period.
- The concept can be expanded considering assumptions for taxes, fees, and other profit and loss influencing cost or revenue drivers.
- Considering the financing costs of the cash inflows and the reinvestment income of the cash outflows, for the AMIRR the invested capital equals the initial investment.

COMPARISON

To discuss and compare the three methods for calculating an MWR, we use the following three sample cash flow streams:

CASH FLOW STREAM 1 (CFS 1)

	31.03.2020	10.04.2020	30.04.2020
BMV	100.00		
CI		100.00	
EMV			185.00

CASH FLOW STREAM 2 (CFS 2)

	31.03.2020	10.04.2020	30.04.2020
BMV	100.00		
CO		50.00	
EMV			46.25

CASH FLOW STREAM 3 (CFS 3)

	31.03.2020	10.04.2020	30.04.2020
BMV	100.00		
CO		50.00	
CI		100.00	
EMV			138.75

The following tables summarize the calculation data for the three methodologies (IRR, MIRR and AMIRR) each for the three scenarios mentioned above.[5]

TABLE 1 – IRR data

IRR	CFS 1	CFS 2	CFS 3
P&L	-15.00	-3.75	-11.25
BMV	100.00	100.00	100.00
EMV	185.00	46.25	138.75
Return	-8.94%	-5.65%	-8.41%
AIC	167.70	66.34	133.82
Adjusted P&L	-8.94	-5.65	-8.41
Adjusted AIC	100.00	100.00	100.00

TABLE 2 – MIRR data

MIRR	CFS 1	CFS 2	CFS 3
P&L	-15.00	-3.75	-11.25
BMV	100.00	100.00	100.00
EMV	185.00	46.25	138.75
Return	-7.44%	-3.62%	-5.49%
AIC	201.66	103.70	204.73
Adjusted P&L	-14.87	-3.62	-11.12
Adjusted AIC	199.87	100.00	202.30

TABLE 3 – AMIRR data

AMIRR	CFS 1	CFS 2	CFS 3
P&L	-15.00	-3.75	-11.25
BMV	100.00	100.00	100.00
EMV	185.00	46.25	138.75
Return	-15.27%	-3.62%	-11.38%
AIC	98.25	103.70	98.82
Adjusted P&L	-15.27	-3.62	-11.38
Adjusted AIC	100.00	100.00	100.00

Comparing the different results, we conclude that:

- Differences in returns are based first on the different reinvestment and financing assumptions and second on the different assumptions on the invested capital,
- IRR and AMIRR assume that the invested capital (adjusted AIC) equals the initial investment, where MIRR assumes that the invested capital equals the present value of all cash inflows, including the initial investment,
- Differences between AMIRR and MIRR only exists due to the treatment of cash inflows within the calculation of the invested capital.

ENDING REMARKS

At the end it should be noted that there is no best method to calculate a "true" MWR. As often, the answer on what measure to use is "it depends". We personally would prefer AMIRR over the IRR as it uses explicit reinvestment and financing assumptions. Whether to use AMIRR or MIRR depends on the question to be answered and whether the funding amount of the cash inflows should be considered in the invested capital. The only question which is left is what finance and reinvestment rates to use. But let there be some space for further thinking and research.

CONTACT INFORMATION

If you have any queries on this article, please contact:

Dr. Stefan J. Illmer:

Tel.: +41 52 620 12 66

Email: stefan.illmer@iipc-ag.com

ENDNOTES

[1] Please see "Decomposing the Money-Weighted Rate of Return" published in the Journal of Performance Measurement, Summer 2003, Volume 7, Number 4, page 42 – 50. An update article "Decomposing the Money-Weighted Rate of Return – an Update" was published by myself in the Journal of Performance Measurement, Fall 2009; Volume 14, Number 1, page 22 – 29.

[2] Please see for example "Performance Attribution – History and Progress" by Carl Bacon published by the CFA Institute Research Foundation in 2019.

[3] It is not intuitive that the IRR is ultimately the return on the initial capital invested. The reason for this is that the financing costs of the cash inflows and the reinvestment income of the cash outflows are intuitively not considered when calculating the invested capital. If this is done, then the profit and loss is adjusted accordingly, and the invested capital is equal to the initial investment.

[4] The oldest article on the MIRR that I have found is Lin, Steven A. Y. (1976). The modified internal rate of return and investment criterion. The Engineering Economist, 21(4), 237–247.

[5] The invested capital (AIC) equals the ratio of the profit and loss (P&L) and the actual return. The adjusted P&L also includes the financing costs of the cash inflows and the reinvestment income of the cash outflows. The adjusted AIC equals the ratio of the adjusted P&L and the actual return. For the MIRR and AMIRR calculation specifically we use a finance and reinvestment rate of 5.00% p.a.