Decomposing the Money-Weighted Rate of Return – an Update

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By Dr. Stefan J. Illmer

INTRODUCTION

In 2003 Wolfgang Marty and I published an article on decomposing the money-weighted rate of return.[1] This article was additional input to the controversy discussion on the two main concepts of return calculation in the investment management industry - time-weighted rate of return (TWR) versus money-weighted rate of return (MWR). Unfortunately this discussion is battled quite emotionally and therefore an industry-wide technical discussion focusing on facts could not really happen yet. Therefore, currently the industry is in the same situation as some years ago where the industry mostly neglected the value of the money-weighted rate of return and of its decomposition.[2]

After the publication of the above mentioned article I focused my research for a while on the critical discussion of the currently used TWR-attribution and based on my findings I developed a methodology for a MWR-attribution. The present article illustrates this methodology and applies it in addition to the absolute profit and loss calculation to also come up with a methodology for a profit and loss attribution.[3]

CALCULATION OF MWR

The starting point for the MWR-attribution is the calculation of the MWR for the portfolio and its benchmark as well as of the portfolio's excess MWR. To calculate the MWR, in the industry different methodologies are used where all but one are approximation methods for the "true" MWR. In the following the internal rate of return methodology (IRR) is used where the IRR is the return that solves the following equation.[4]

$$\frac{EMV}{(1+IRR)^T} + \left(\sum_{t=1}^{T-1} \frac{-C_t}{(1+IRR)^t} \right) - BMV = 0 \qquad (1)$$

with

IRR = Internal rate of return,
BMV = Beginning market value,
EMV = Ending market value at T,
C_t = Cash flow at t,
T = Time of the end of the measurement period,
t = Time of cash flow.

The IRR of the portfolio is therefore defined by the respective market values at the beginning and the end of the measurement period and by the respective cash flows during the measurement period.

$$\frac{EMV_P}{(1+IRR_P)^T} + \left(\sum_{t=1}^{T-1} \frac{-C_{P,t}}{(1+IRR_P)^t} \right) - BMV_P = 0 \qquad (2)$$

with

IRR_P = IRR of portfolio,
BMV_P = Portfolio beginning market value,
EMV_P = Portfolio ending market value at T,
$C_{P,t}$ = Portfolio cash flow at t.

Similarly the IRR for the benchmark is defined by the respective market values at the beginning and the end of the measurement period and by the respective cash flows during the measurement period.[5] Here it is important that the cash inflows (outflows) are invested (de-invested) according to the actual benchmark asset allocation at the time of the cash flow and that the returns of the money invested equal the respective returns of the underlying benchmark investments.

$$\frac{EMV_B}{(1+IRR_B)^T} + \left(\sum_{t=1}^{T-1} \frac{-C_{B,t}}{(1+IRR_B)^t} \right) - BMV_B = 0 \qquad (3)$$

with

IRR_B = IRR of benchmark,
BMV_B = Benchmark beginning market value,
EMV_B = Benchmark ending market value at T,
$C_{B,t}$ = Benchmark cash flow at t.

Given the IRR of the portfolio and the benchmark, the portfolio's excess IRR can be calculated using the formula below.

$$EIRR_P = IRR_P - IRR_B \quad (4)$$

with

$EIRR_P$ = Excess IRR.

CONTRIBUTION TO MWR

The second step towards the MWR-attribution is the calculation of the contributions to MWR for the portfolio and its benchmark as well as the contributions to the portfolio's excess MWR.

To calculate the contributions to IRR it is important to understand the concept of the internal rate of return and its underlying assumptions. The contributions to IRR are not based on the weights of the asset classes at the beginning of the measurement period but instead based on the average invested capital which is especially relevant if there are interim cash flows within the measurement period.[6] Considering that the IRR is the return that determines the absolute profit and loss (P&L), the average invested capital can be derived using the following formulas.

$$PL_P = EMV_P - BMV_P - \sum_{t=1}^{T-1} C_{P,t} \quad (5)$$

with

PL_P = P & L of portfolio.

$$\begin{aligned} PL_P &= \sum_{i=1}^{n} PL_{P,i} \\ &= \sum_{i=1}^{n} \left(EMV_{P,i} - BMV_{P,i} - \sum_{t=1}^{T-1} C_{P,i,t} \right) \end{aligned} \quad (6)$$

with

$PL_{P,i}$ = P & L of asset class i,
$EMV_{P,i}$ = Ending market value of asset class i,
$BMV_{P,i}$ = Beginning market value of asset class i,
$C_{P,i,t}$ = Cash flow of asset class i at t,
n = Number of asset classes.

$$AIC_P = \frac{PL_P}{IRR_P} \quad (7)$$

with

AIC_P = Average invested capital of portfolio.

$$AIC_{P,i} = \frac{PL_{P,i}}{IRR_{P,i}} \quad (8)$$

with

$AIC_{P,i}$ = Average invested capital of asset class i,
$IRR_{P,i}$ = IRR of asset class i.

It is important to note that the average invested capital of the total portfolio does not have to be equal to the sum of the average invested capitals of all asset classes.[7] As stated below, depending on the underlying data the average invested capital of the total portfolio can be bigger, equal or less than the sum of the average invested capitals of all asset classes.

$$AIC_P \leq \text{or} = \text{or} \geq \sum_{i=1}^{n} AIC_{P,i} \quad (9)$$

Using the above mentioned formulas the IRR of the portfolio can be expressed as the sum of the return contributions of each asset class.

$$\begin{aligned} IRR_P &= \frac{PL_P}{AIC_P} \\ &= \sum_{i=1}^{n} \frac{PL_{P,i}}{AIC_P} \\ &= \sum_{i=1}^{n} \frac{AIC_{P,i}}{AIC_P} \times IRR_{P,i} \\ &= \sum_{i=1}^{n} RC_{P,i} \end{aligned} \quad (10)$$

with

$RC_{P,i}$ = Return contribution of asset class i.

Formulas (5) to (10) cover the calculations for the portfolio but these formulas can also be applied for the benchmark by just substituting the index P by B like in formula (11).

$$
\begin{aligned}
IRR_B &= \frac{PL_B}{AIC_B} \\
&= \sum_{i=1}^{n} \frac{PL_{B,i}}{AIC_B} \\
&= \sum_{i=1}^{n} \frac{AIC_{B,i}}{AIC_B} \times IRR_{B,i} \\
&= \sum_{i=1}^{n} RC_{B,i}
\end{aligned}
\qquad (11)
$$

with

$RC_{B,i}$ = Return contribution of asset class i,
PL_B = P & L of benchmark,
$PL_{B,i}$ = P & L of asset class i,
AIC_B = Average invested capital of benchmark,
$AIC_{B,i}$ = Average invested capital of asset class i.

The contributions to the portfolio's excess IRR can be calculated using the following formula.

$$
\begin{aligned}
EIRR_P &= IRR_P - IRR_B \\
&= \sum_{i=1}^{n} RC_{P,i} - \sum_{i=1}^{n} RC_{B,i} \\
&= \sum_{i=1}^{n} \frac{AIC_{P,i}}{AIC_P} \times IRR_{P,i} - \sum_{i=1}^{n} \frac{AIC_{B,i}}{AIC_B} \times IRR_{B,i} \\
&= \sum_{i=1}^{n} \frac{PL_{P,i}}{AIC_P} - \sum_{i=1}^{n} \frac{PL_{B,i}}{AIC_B}
\end{aligned}
\qquad (12)
$$

MWR-ATTRIBUTION

The last step towards MWR-attribution is the decomposition of the portfolio's excess MWR in a way that it reflects the different steps of the decision making process. Here the excess IRR is decomposed according to the return attribution methodology published by Brinson, Hood and Beebower in 1986 and therefore split up into the three management effects: asset allocation effect, stock picking effect and interaction effect.[8]

$$
\begin{aligned}
EIRR_P &= IRR_P - IRR_B \\
&= AAE_P + SPE_P + IAE_P \\
&= \sum_{i=1}^{n} AAE_{P,i} + \sum_{i=1}^{n} SPE_{P,i} + \sum_{i=1}^{n} IAE_{P,i}
\end{aligned}
\qquad (13)
$$

with

AAE_P = Asset allocation effect of portfolio,
SPE_P = Stock picking effect of portfolio,
IAE_P = Interaction effect of portfolio,
$AAE_{P,i}$ = Asset allocation effect of asset class i,
$SPE_{P,i}$ = Stock picking effect of asset class i,
$IAE_{P,i}$ = Interaction effect of asset class i.

To calculate the different management effects the framework for analyzing portfolio returns by Brinson, et al. (1986) is used.[9] Table 1 illustrates this framework applied to the MWR-attribution.

Table 1 Simple framework for MWR-attribution

Asset Allocation	Selection: Actual	Selection: Passive
Actual	**Quadrant IV** IRR of actual portfolio	**Quadrant II** IRR of notional portfolio 1 => active asset allocation portfolio
Passive	**Quadrant III** IRR of notional portfolio 2 => active stock picking portfolio	**Quadrant I** IRR of benchmark

Quadrant I represents the IRR of the benchmark where for the calculation all (de-)investments made in each asset class – based on the passive weights of the asset class and expressed as cash flows - and the passive index returns for each asset class are needed.

Quadrant II represents the IRR of the notional portfolio 1 which reflects the active asset allocation of the portfolio assuming no stock picking. To calculate the IRR of the notional portfolio 1 all (de-)investments made in each asset class – based on the actual weights of the asset class and expressed as cash flows - and the passive index returns for each asset class are needed.

Quadrant III represents the IRR of the notional portfolio 2 which reflects the active stock picking of the portfolio assuming no active asset allocation. To calculate the

IRR of the notional portfolio 2 all (de-)investments made in each asset class – based on the passive weights of the asset class and expressed as cash flows - and the actual returns for each asset class are needed.

Quadrant IV represents the IRR of the portfolio where for the calculation all (de-)investments made in each asset class – based on the actual weights of the asset class and expressed as cash flows - and the actual returns for each asset class are needed.

Based on the aforementioned framework for MWR-attribution the different management effects can be calculated as follows.

$$\begin{aligned} AAE_P &= \text{QuadrantII} - \text{QuadrantI} \\ &= IRR_{NP1} - IRR_B \\ &= \sum_{i=1}^{n} RC_{NP1,i} - \sum_{i=1}^{n} RC_{B,i} \end{aligned} \quad (14)$$

with

IRR_{NP1} = IRR of notional portfolio 1,
$RC_{NP1,i}$ = Return contribution of asset class i.

$$\begin{aligned} SPE_P &= \text{QuadrantIII} - \text{QuadrantI} \\ &= IRR_{NP2} - IRR_B \\ &= \sum_{i=1}^{n} RC_{NP2,i} - \sum_{i=1}^{n} RC_{B,i} \end{aligned} \quad (15)$$

with

IRR_{NP2} = IRR of notional portfolio 2,
$RC_{NP2,i}$ = Return contribution of asset class i.

$$\begin{aligned} IAE_P &= \begin{array}{l} \text{QuadrantIV} - \text{QuadrantIII} \\ - \text{QuadrantII} + \text{QuadrantI} \end{array} \\ &= \begin{array}{l} IRR_P - IRR_{NP2} \\ - IRR_{NP1} + IRR_B \end{array} \\ &= \begin{array}{l} \sum_{i=1}^{n} RC_{P,i} - \sum_{i=1}^{n} RC_{NP2,i} \\ - \sum_{i=1}^{n} RC_{NP1,i} + \sum_{i=1}^{n} RC_{B,i} \end{array} \end{aligned} \quad (16)$$

The formulas (14) to (16) can be used as well to calculate the different management effects on an asset class level.

$$AAE_{P,i} = RC_{NP1,i} - RC_{B,i} \quad (17)$$

$$SPE_{P,i} = RC_{NP2,i} - RC_{B,i} \quad (18)$$

$$IAE_{P,i} = RC_{P,i} - RC_{NP2,i} - RC_{NP1,i} + RC_{B,i} \quad (19)$$

A property of the IRR is that the (percentage) returns can be transformed into the observable absolute profit and loss figures per unit of currency.

Just as for the IRR, the profit and loss of the portfolio and of the asset classes are determined by the respective market values at the beginning and the end of the measurement period and by the respective cash flows during the measurement period and can be calculated using formulas (5) and (6). Formulas (5) to (6) can also be applied for the benchmark by just substituting the index P by B.

$$PL_B = EMV_B - BMV_B - \sum_{t=1}^{T-1} C_{B,t} \quad (20)$$

$$\begin{aligned} PL_B &= \sum_{i=1}^{n} PL_{B,i} \\ &= \sum_{i=1}^{n} \left(EMV_{B,i} - BMV_{B,i} - \sum_{t=1}^{T-1} C_{B,i,t} \right) \end{aligned} \quad (21)$$

with

$EMV_{B,i}$ = Ending market value of asset class i,
$BMV_{B,i}$ = Beginning market value of asset class i,
$C_{B,i,t}$ = Cash flow of asset class i at t.

Given the profit and loss of the portfolio and the benchmark, the portfolio's excess profit and loss and its contributions can be calculated using the below formula.

$$\begin{aligned} EPL_P &= PL_P - PL_B \\ &= \sum_{i=1}^{n} PL_{P,i} - \sum_{i=1}^{n} PL_{B,i} \end{aligned} \quad (22)$$

with

EPL_P = Excess profit and loss.

Table 2 Simple framework for profit and loss attribution

		Selection: Actual	Selection: Passive
Actual		**Quadrant IV** P&L of actual portfolio	**Quadrant II** P&L of notional portfolio 1 => active asset allocation portfolio

PROFIT AND LOSS ATTRIBUTION

Asset Allocation		
Passive	**Quadrant III** P&L of notional portfolio 2 => active stock picking portfolio	**Quadrant I** P&L of benchmark

As shown in table 2 the aforementioned framework for MWR-attribution can also be applied to the profit and loss attribution to decompose the excess profit and loss into the three above mentioned management effects on total portfolio and asset class level.

$$\begin{aligned} EPL_P &= PL_P - PL_B \\ &= AAPL_P + SPPL_P + IAPL_P \\ &= \sum_{i=1}^{n} AAPL_{P,i} + \sum_{i=1}^{n} SPPL_{P,i} + \sum_{i=1}^{n} IAPL_{P,i} \end{aligned} \quad (23)$$

with

$AAPL_P$ = P & L due to asset allocation of portfolio,
$SPPL_P$ = P & L due to stock picking of portfolio,
$IAPL_P$ = P & L due to interaction effect of portfolio,
$AAPL_{P,i}$ = P & L due to asset allocation of asset class i,
$SPPL_{P,i}$ = P & L due to stock picking of asset class i,
$IAPL_{P,i}$ = P & L due to interaction of asset class i.

$$\begin{aligned} AAPL_P &= PL_{NP1} - PL_B \\ &= \sum_{i=1}^{n} PL_{NP1,i} - \sum_{i=1}^{n} PL_{B,i} \end{aligned} \quad (24)$$

with

PL_{NP1} = P & L of notional portfolio 1,
$PL_{NP1,i}$ = P & L of asset class i.

$$\begin{aligned} SPPL_P &= PL_{NP2} - PL_B \\ &= \sum_{i=1}^{n} PL_{NP2,i} - \sum_{i=1}^{n} PL_{B,i} \end{aligned} \quad (25)$$

with

PL_{NP2} = P & L of notional portfolio 2,
$PL_{NP2,i}$ = P & L of asset class i.

$$\begin{aligned} IAPL_P &= PL_P - PL_{NP2} \\ &\quad - PL_{NP1} + PL_B \\ &= \sum_{i=1}^{n} PL_{P,i} - \sum_{i=1}^{n} PL_{NP2,i} \\ &\quad - \sum_{i=1}^{n} PL_{NP1,i} + \sum_{i=1}^{n} PL_{B,i} \end{aligned} \quad (26)$$

$$AAPL_{P,i} = PL_{NP1,i} - PL_{B,i} \quad (27)$$

$$SPPL_{P,i} = PL_{NP2,i} - PL_{B,i} \quad (28)$$

$$IAE_{P,i} = PL_{P,i} - PL_{NP2,i} - PL_{NP1,i} + PL_{B,i} \quad (29)$$

AN EXAMPLE FOR A MWR-ATTRIBUTION AND A PROFIT AND LOSS ATTRIBUTION

In this section an example for a MWR-Attribution and a profit and loss attribution is illustrated by considering a multi-asset class portfolio for the two year period 31.12.2006 until 31.12.2008 which is invested in two asset classes (A and B) and which is rebalanced on a yearly basis at the beginning of the year. This portfolio is managed against a benchmark which is also invested in these two asset classes and rebalanced on a yearly basis at the beginning of the year.

At the beginning of 2007 EUR 150 and at the beginning of 2008 additional EUR 100 are invested into the portfolio according to the then current active asset allocation and stock pickings. The (passive and actual) returns and the (passive and actual) weights of the portfolio and the benchmark are stated in table 3.[10] To be able to calculate the different returns and return contributions, the weights have to transferred into cash flows like for example for the actual portfolio where the cash inflow into asset class A of EUR 75 is 50% of the initial cash inflows of EUR 150 into the portfolio or the cash outflow at the beginning of 2008 of EUR 47.6 which is the difference between the market value invested in asset class A at the end of 2007 of EUR 86.3 minus 15% of the market value at the end of 2007 of EUR 257.5 (new weight of asset class A at the beginning of 2008).[11]

Table 3 shows that the IRR of the portfolio is 13.8% and the IRR of the benchmark is 0.1% what causes an excess IRR of 13.7%.[12] Comparing the market values at the end of 2008 of the portfolio and of the benchmark leads to the fact that the portfolio manager generated an excess profit of EUR 27.2.

Table 4 summarizes the results of the IRR-attribution and the profit and loss attribution which are based on the return and the profit and loss data of the portfolio, the benchmark, the notional portfolio 1 and the notional portfolio 2. At the bottom, table 4 contains the detailed

results for the IRR-attribution showing that the total excess IRR is 13.7% with a total asset allocation effect of -7.2%, a total stock picking effect of 7.8% and a total interaction effect of 13.3%.[13] Exemplary, around 2/3 of the asset allocation effect is due to the investments in asset class A (-4.9%) but around 2/3 of the stock picking is due to the investments made in asset class B (5.4%). Furthermore table 4 contains the result of the profit and loss attribution showing that the total excess profit and loss is EUR 27.2 with a total asset allocation

Table 3 Example for MWR-attribution and profit and loss attribution – underlying data

Actual Portfolio (IRR)			
	Period 1	**Period 2**	
Dates	31.12.2006	31.12.2007	31.12.2008
	Cash flow at beginning of period	Cash flow at beginning of period	Market value at the end of period
Asset A	-75.0	47.6	36.7
Asset B	-75.0	-147.6	240.8
Portfolio	-150.0	-100.0	277.5
	Actual weights at beginning of period	Actual weights at beginning of period	Weights at the end of period
Asset A	**50.0%**	**15.0%**	13.2%
Asset B	**50.0%**	**85.0%**	86.8%
Portfolio	100.0%	100.0%	100.0%
	Actual return	Actual return	Cummulative return
Asset A	**15.0%**	**-5.0%**	17.9%
Asset B	**-5.0%**	**10.0%**	12.4%
Portfolio	5.0%	7.8%	13.8%

Notional Portfolio 1 (IRR)			
	Period 1	**Period 2**	
Dates	31.12.2006	31.12.2007	31.12.2008
	Cash flow at beginning of period	Cash flow at beginning of period	Market value at the end of period
Asset A	-75.0	23.6	40.0
Asset B	-75.0	-123.6	195.8
Portfolio	-150.0	-100.0	235.8
	Actual weights at beginning of period	Actual weights at beginning of period	Weights at the end of period
Asset A	**50.0%**	**15.0%**	17.0%
Asset B	**50.0%**	**85.0%**	83.0%
Portfolio	100.0%	100.0%	100.0%
	Passive return	Passive return	Cummulative return
Asset A	**-20.0%**	**10.0%**	-18.2%
Asset B	**10.0%**	**-5.0%**	-2.0%
Portfolio	-5.0%	-2.8%	-7.0%

Notional Portfolio 2 (IRR)			
	Period 1	**Period 2**	
Dates	31.12.2006	31.12.2007	31.12.2008
	Cash flow at beginning of period	Cash flow at beginning of period	Investment at the end of period
Asset A	-45.0	-23.7	71.7
Asset B	-105.0	-76.3	193.7
Portfolio	-150.0	-100.0	265.3
	Passive weights at beginning of period	Passive weights at beginning of period	Weights at the end of period
Asset A	**30.0%**	**30.0%**	27.0%
Asset B	**70.0%**	**70.0%**	73.0%
Portfolio	100.0%	100.0%	100.0%
	Actual return	Actual return	Cummulative return
Asset A	**15.0%**	**-5.0%**	5.2%
Asset B	**-5.0%**	**10.0%**	8.7%
Portfolio	1.0%	5.5%	7.7%

Benchmark (IRR)			
	Period 1	**Period 2**	
Dates	31.12.2006	31.12.2007	31.12.2008
	Cash flow at beginning of period	Cash flow at beginning of period	Market value at the end of period
Asset A	-45.0	-39.5	83.0
Asset B	-105.0	-60.6	167.2
Portfolio	-150.0	-100.0	250.2
	Passive weights at beginning of period	Passive weights at beginning of period	Weights at the end of period
Asset A	**30.0%**	**30.0%**	33.2%
Asset B	**70.0%**	**70.0%**	66.8%
Portfolio	100.0%	100.0%	100.0%
	Passive return	Passive return	Cummulative return
Asset A	**-20.0%**	**10.0%**	-2.2%
Asset B	**10.0%**	**-5.0%**	1.3%
Portfolio	1.0%	-0.5%	0.1%

Table 4 Example for MWR-attribution and profit and loss attribution – attribution details

Actual Portfolio (IRR)					Notional Portfolio 1 (IRR)				
	P&L	AIC	IRR	RC		P&L	AIC	IRR	RC
Asset A	9.3	52.1	17.9%	4.7%	Asset A	-11.4	62.6	-18.2%	-5.7%
Asset B	18.1	146.8	12.4%	9.1%	Asset B	-2.8	137.2	-2.0%	-1.4%
Portfolio	27.5	198.4	13.8%	13.8%	Portfolio	-14.2	201.0	-7.0%	-7.0%

Notional Portfolio 2 (IRR)					Benchmark (IRR)				
	P&L	AIC	IRR	RC		P&L	AIC	IRR	RC
Asset A	3.0	56.7	5.2%	1.5%	Asset A	-1.5	64.9	-2.2%	-0.7%
Asset B	12.4	142.4	8.7%	6.2%	Asset B	1.7	135.2	1.3%	0.8%
Portfolio	15.3	199.1	7.7%	7.7%	Portfolio	0.2	200.1	0.1%	0.1%

IRR-Attribution					Profit and Loss Attribution				
	AAE	SPE	IAE	Total		AAPL	SPPL	IAPL	Total
Asset A	-4.9%	2.2%	8.1%	5.4%	Asset A	-9.9	4.4	16.2	10.8
Asset B	-2.2%	5.4%	5.2%	8.3%	Asset B	-4.5	10.7	10.3	16.4
Excess	-7.2%	7.6%	13.3%	13.7%	Excess	-14.4	15.1	26.5	27.2

effect of EUR -14.4, a total stock picking effect of EUR 15.1 and a total interaction effect of EUR 26.5. Again around 2/3 of the asset allocation effect is due to the investments in asset class A (EUR -9.9) but around 2/3 of the stock picking is due to the investments made in asset class B (EUR 10.7).

CONCLUSION

This article illustrates two methodologies which help to better understand where the historical return (expressed as an IRR) and where the historical profit and loss is coming from. The MWR-attribution has a lot of advantages as its results are in line with the profit and loss attribution and as it correctly contributes the timing effect to the asset allocation and stock picking effect. This is additional evidence that the MWR concept still adds value and should be considered if analyzing the performance of investment management activities.

CONTACT INFORMATION

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ENDNOTES

1 "Decomposing the Money-Weighted Rate of Return"; in: Journal of Performance Measurement; Summer 2003, pages 42-50; by Dr. Stefan J. Illmer and Wolfgang Marty.

2 For a detailed discussion on the two main concepts of return calculation as well as on the concept of performance attribution please be referred to textbooks on performance measurement and performance attribution from authors like Carl Bacon (Practical Portfolio Performance Measurement and Attribution), Bruce Feibel (Investment Performance Measurement), David Spaulding (The Handbook of Investment Performance: A User's Guide) and others.

3 This article is based on the presentation "Investigating the Case for MWR-Attribution" held by Dr. Stefan J. Illmer at the PMAR V conference in Philadelphia 2007. The presentation is available from the author (see contact information).
For the differences between performance attribution and return as well risk attribution, please see "EIPC Guidance on Performance Attribution Presentation: A Step Towards Standardization of Performance Attribution"; in: Journal of

Performance Measurement; Summer 2004; pages 46-62; by Dr. Stefan J. Illmer and Dimitri Senik.

The author is aware of the underlying assumptions of the IRR and the shortcomings if calculating the IRR. To focus more on the methodology it should be up to other articles to discuss these critics and to provide respective solutions. Furthermore it should also be up to other articles to compare the MWR-attribution with the TWR-attribution and to discuss the consequences on the "historical" risk measurement and risk attribution as well as on the whole performance monitoring process.

[4] Internal rate of return (IRR) is a MWR and is called the "true" MWR as it is not only the most precise method for calculating a MWR but the one methodology that solves the full calculation problem. The IRR is the return / interest rate that causes the ending market value and intermediate cash flows to be discounted to the beginning market value.

In the following MWR and IRR as well as MWR-attribution and IRR-attribution are therefore used interchangeable.

[5] The market value of the benchmark at the beginning of the measurement period equals the respective market value of the portfolio at the beginning of the measurement period and is fictitiously invested in the different benchmark indices according to the benchmark asset allocation weights. The market value of the benchmark at the end of the measurement period is then determined by the benchmark weights, the rebalancing activities, the (de-) investment of the interim cash flows and the respective index returns over time.

[6] In the following the term „asset class" is used instead of using the broader term „segments".

[7] This property is difficult to follow but is correct as it is based on the underlying reinvestment assumption of the IRR methodology.

[8] „Determinants of Portfolio Performance"; in: Financial Analysts Journal; July/August 1986; pages 39-44; by G. Brinson, R. Hood and G. Beebower.

[9] The MWR-attribution and profit and loss attribution can be adjusted to reflect other concepts for decomposing portfolio returns. The author is already working on this and is willing to share his insights with interested readers (see contact information).

[10] The spreadsheet with the calculations is available from the author (see contact information).

[11] The market value invested in asset class A at the end of 2007 is defined by the money invested at the beginning of 2007 and its return:
$86.25 = 75 \times (1 + 15\%)$.

In table 3 a negative sign in front of the cash flows indicate cash inflows (buy) and a positive sign in front of the cash flows indicate a cash outflows (sell). This helps with the calculation of the IRR where cash inflows have always a negative sign and vice versa.

In table 3 the cumulative returns and return contributions are not annualized. If needed the calculation can be transferred to annualized returns by annualizing the IRRs where it is important to choose a consistent date convention.

[12] This assumes that the portfolio manager had discretion over the cash inflow at the beginning of 2008. Otherwise the calculation has to be adjusted in a way that the timing effect of the cash inflow has to be separated following the methodology explained in the article "Decomposing the Money-Weighted Rate of Return" mentioned in the endnote 1.

[13] The interaction effect of 13.3% is quite big but due to the extreme example.