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New and less common ways of measuring returns

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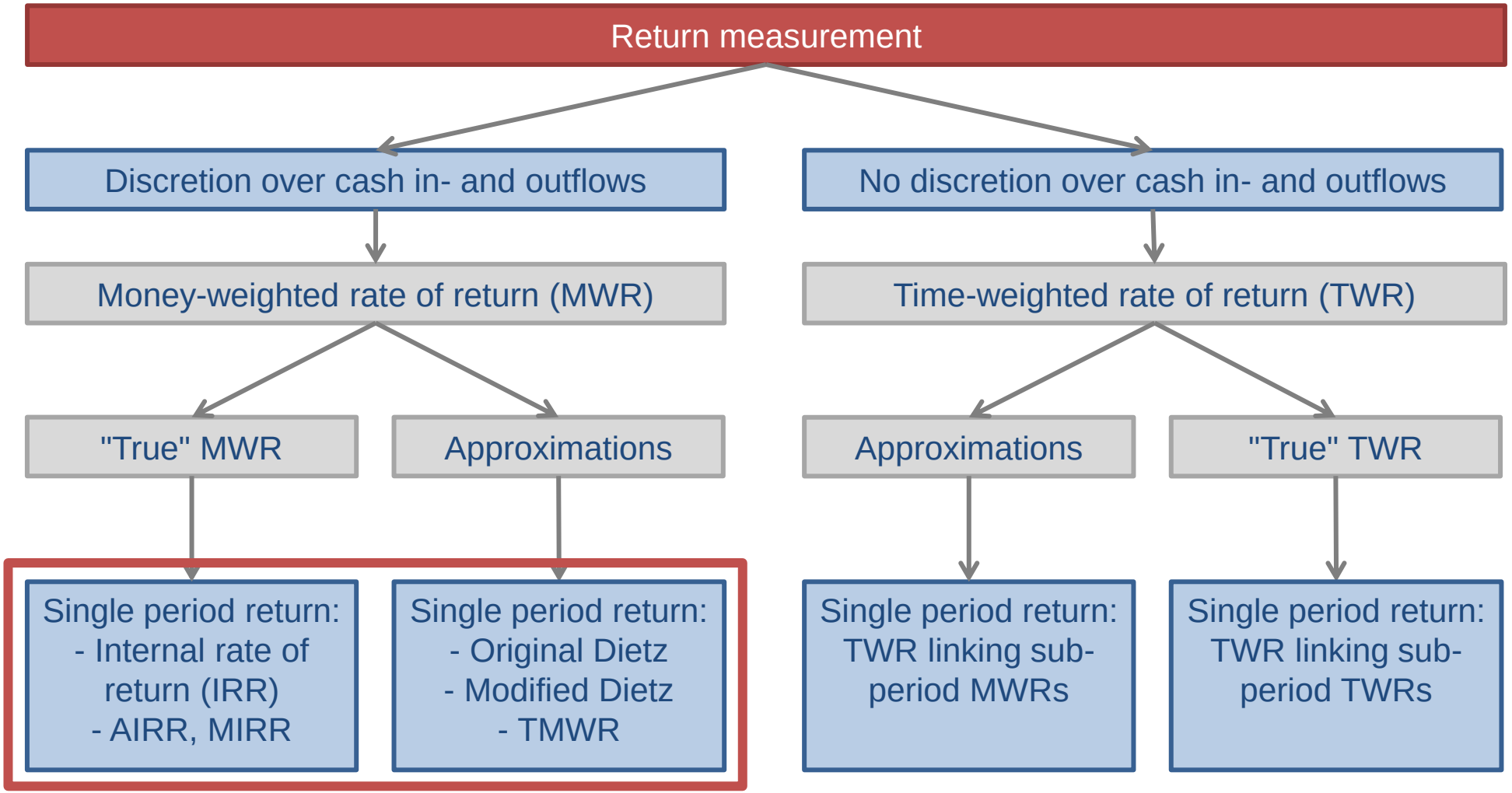
Return measurement – The big picture (1/2)



Return measurement – The big picture (2a/2)

A rate of return is the benefit one received from an investment over a period of time expressed as a percentage - where the benefit covers income as well as capital gains or losses. In general the return is the ratio of profit and loss to invested capital or capital at risk.

$$R = \frac{Profit\ and\ loss}{Capital} = \frac{EMV - BMV - Cif + Cof}{BMV}$$

R = *Rate of return.*
EMV = *Ending market value.*
BMV = *Beginning market value.*
Cif = *External cash inflow.*
Cof = *External cash outflow.*

Important: It seems to be easy but in reality it is not!

Return measurement – The big picture (2b/2)

Considering a portfolio of investments the generic formula for the calculation of the rate of return can be further split up to illustrate the different components relevant for the actual rate of return – if aggregating from an asset segment level.

$$R_P = \frac{Profit\ and\ loss}{Capital} = \frac{PL_P}{AIC_P} = \frac{\sum_{i=1}^{n} PL_i}{AIC_P} = \frac{\sum_{i=1}^{n} AIC_i \times R_i}{AIC_{Total}}$$

R_P	$=$ *Rate of return of total portfolio.*
R_i	$=$ *Rate of return of asset segment i.*
PL_P	$=$ *Profit and loss of total portfolio.*
PL_i	$=$ *Profit and loss of asset segment i.*
AIC_i	$=$ *Average invested capital of asset segment i.*
AIC_{Total}	$=$ *Average invested capital of total portfolio.*

Return measurement – The big picture (2c/2)

The complexity with the return measurement arises not from the calculation of the profit and loss but from the definition of the (average) invested capital and how this should be calculated – especially if cash flows occurred. Furthermore return measurement methodologies differ if not only a single but multiple sub-periods are considered and if cumulating or compounding the sub-period returns.

Internal rate of return (IRR)

IRR – Definition (1a/4)

- Internal rate of return (IRR) is a money-weighted rate of return (MWR) and is called a "true" MWR as it is a precise method for calculating a MWR and no approximation of the MWR.
- The IRR is the (average) return or interest rate that causes the ending value and the intermediate external cash flows to be discounted to the beginning value.
- Due to its specific compounding characteristics, the IRR is also the (average) return or interest rate that causes the beginning value and intermediate cash flows to grow to the ending value.
- The IRR is the average annual rate of return that will be earned if the external cash flows are financed or reinvested using an implicit reinvestment assumptions.

IRR – Definition (1b/4)

- The implicit reinvestment assumption of the IRR methodology with respect to the interim external cash flows is that:
  1) Cash inflows are financed at an interest rate (finance rate) – maybe seen as the cost of capital – that is identical to the IRR.
  2) Cash outflows are reinvested at an interest rate (reinvestment rate) – maybe seen as the cost of capital – that is identical to the IRR.

IRR – Formula (2/4)

$$0 = \frac{EMV_T}{(1+IRR)^{Y_T}} + \left(\sum_{t=1}^{T-1} \frac{-CF_t}{(1+IRR)^{Y_{t-0}}} \right) - BMV$$

BMV = *Beginning market value.*
EMV_T = *Ending market value at T.*
IRR = *Internal rate of return (annualized).*
CF_t = *Cash flow at t.*
Y_T = *Length of measurement period (to be measured in years - 365).*
Y_{t-0} = *Length of time period between the beginning of the measurement period and the date of the cash flow (to be measured in years - 365).*

IRR – Example (3/4)

Portfolio data	31.12.2010	31.03.2011	30.06.2011	30.09.2011	31.12.2011	31.03.2012
Assets	100.0	105.0	111.3	86.8	85.1	80.9
Cash flows (end of sub-period)		0.0	0.0	-20.0	0.0	0.0
IRR cash flow stream	-100.0	0.0	0.0	20.0	0.0	80.9
TWRs		5.00%	6.00%	-4.00%	-2.00%	-5.00%
Annualized total period IRR	0.74%					
Annualized total period TWR	-0.42%					

IRR – Critics (4a/4)

- IRR is a money-weighted rate of return as it is influenced by the timing of cash flows – therefore may not useful to assess a portfolio manager without discretion over the cash in- and outflows.
- Implicit interim profit and loss is not equal to the profit and loss observed on the balance sheet.
- IRR is an average rate of return not based on the actual interim values but on the actual cash flows.
- IRR is not easy to calculate.
- IRR imply unrealistic (re-) investment assumptions - means the IRR reinvestment rate is a result of the cash flows and their timing.
- IRR is path-depending with respect to the cash flows.
- IRR can not always be calculated and is sometimes not unique – means has an issue with multiple solutions.

IRR – Critics (4b/4)

- IRR is useful for both ex ante investment decision-making and ex post performance evaluation.

Time- & money-weighted rate of return (TMWR)

TMWR – Starting point (1/5)

- No standard methodology to calculate returns in the investment management industry.
- Common practice to calculate TWR and MWR depending on the type of investment and on the perspective (investor versus portfolio manager).
- Search for a measure that combines the advantages of the TWR and those of the MWR.

TMWR – Definition (2/5)

- Time- & money-weighted rate of return is a combination of the periodic return feature of the TWR and the capital timing and weighting attributes of the IRR.
- TMWR concept is to calculate sub-period TWRs and to value or money weight these sub-period TWRs over the entire measurement period, to finally derive a cumulative return that is then annualized.
- In comparison the TWR equally weights the sub-period returns – where the reinvestment rates are the respective sub-period TWRs.
- In comparison the IRR uses a reinvestment rate what is equal to the overall IRR – with money weighting of the sub-period returns.
- TMWR money weights the sub-period returns – where the reinvestment rates are the respective sub-period TWRs.
- TMWR uses the same sub-period returns than the TWR but weights and annualizes them differently.

TMWR – Formula (3a/5)

- TMWR is money-weighting the sub-period returns where the weights are based on actual values and not on IRR implied interim values.
- TMWR is calculated as:

$$TMWR = \sum_{i=1}^{n} w_i \times r_i$$

where:

$$w_i = \frac{AIC_i}{\sum_{i=1}^{n} AIC_i}$$

w_i = *Money weight for period i.*
r_i = *TWR of period i.*
AIC_i = *Average invested capital for period i.*
n = *Number of periods.*

TMWR – Formula (3b/5)

- Average invested capital is calculated as:

$$AIC_i = BMW_i + \sum_{i=1}^{n} w_i \times CF_i$$

$$where: \; w_i = \frac{D - D_i}{D}$$

- Annualized TMWR is calculated as:

$$TMWR_{ann} = (1 + TMWR)^y - 1$$

D = *Number of calendar days in period.*
D_i = *Day of cash flow i.*
y = *Number of sub-periods in a year.*

TMWR – Example (4/5)

Portfolio data	31.12.2010	31.03.2011	30.06.2011	30.09.2011	31.12.2011	31.03.2012
Assets	100.0	105.0	111.3	86.8	85.1	80.9
Cash flows (end of sub-period)		0.0	0.0	-20.0	0.0	0.0
IRR cash flow stream	-100.0	0.0	0.0	20.0	0.0	80.9
TWRs		5.00%	6.00%	-4.00%	-2.00%	-5.00%
TMWR sub-period AIC		100.0	105.0	111.3	86.8	85.1
TMWR sum of sub-period AIC	488.3					
TMWR sub-period weights		20.48%	21.50%	22.80%	17.79%	17.43%
TMWR weighted TWRs		1.02%	1.29%	-0.91%	-0.36%	-0.87%
Annualized total period IRR	0.74%					
Annualized total period TWR	-0.42%					
Annualized total period TMWR	0.70%					

TMWR – Critics (5a/5)

- TMWR not equal to IRR or TWR if there are no cash flows.
- TMWR is not a precise method for calculating a MWR and therefore an approximation of the MWR.
- TMWR sub-period AIC is an approximation of the true or economical AIC.
- TMWR total period AIC do not consider the timing of the cash flows (no cost of capital).
- TMWR seems to be nearer to the IRR than to the TWR but differences to the IRR can be considerable.
- TMWR contribution or decomposition not addressed yet.
- TMWR is a money-weighted rate of return as it is influenced by the timing of cash flows – therefore may not useful to assess a portfolio manager without discretion over the cash in- and outflows.

TMWR – Critics (5b/5)

- Implicit interim profit and loss equals to the profit and loss observed on the balance sheet – because implicit assumption that cost of capital equals 0.00%.
- TMWR can always be calculated and is unique – means has no issue with multiple solutions.
- TMWR is easy to calculate and may be used as a (first) estimation to the IRR.

Average internal rate of return (AIRR)

AIRR – Starting point (1/6)

- Question: What is the IRR a rate of return on:
  - The initial contribution?
  - All contributions?
  - All contributions net of all distributions?
  - None of the above?
- Interim values implied by the IRR differ from the true interim values.
- IRR solution might not be unique - multiple solutions.
- IRR equation might not be solvable.

AIRR – Definition (2a/6)

- Average internal rate of return (AIRR) is a money-weighted rate of return and is called a "true" MWR as it is a precise method for calculating a MWR and no approximation of the MWR.
- AIRR is the average of the periodic rate of returns - weighted according to the net present value (NPV) of the interim values - where each is the period's profit and loss (income and capital gains) as percentage of the respective interim value.
- AIRR is money-weighting the sub-period returns where the weights are based on actual interim values and not on IRR implied interim values.
- AIRR equals the IRR (but nearly never)
  - if the interim values are equal to the implied IRR interim values or
  - if the implied IRR interim values are different from the actual ones but the cost of capital equals the IRR.

AIRR – Definition (2b/6)

- AIRR is different from IRR if the implied IRR interim values are different from the actual ones and the costs of capital are different from the computed IRR.

AIRR – Formula

(3a/6)

$$AIRR = \frac{\sum_{i=1}^{n} r_i \times PV(V_{i-1})}{\sum_{i=1}^{n} PV(V_{i-1})}$$

where

$$r_i = \frac{PL_i}{V_{i-1}}$$

$$PL_i = CF_i + V_i - V_{i-1}$$

$$PV(V_{i-1}) = \frac{V_{i-1}}{(1 + CoC)^{Y_{t-0}}}$$

AIRR – Formula (3b/6)

r_i	=	*TWR for sub−period i.*
$PV(V_{i-1})$	=	*Present value of the interim value at the end of sub-period i-1 or at the beginning of sub-period i.*
PL_i	=	*Profit and loss for sub-period i.*
CF_i	=	*Cash flow during the sub-period i.*
V_i	=	*Interim value at the end of sub-period i or beginning of sub-period i+1.*
V_{i-1}	=	*Interim value at the end of sub-period i-1 or beginning of sub-period i.*
n	=	*Number of periods.*
CoC	=	*Rate for cost of capital (annualized).*
Y_{t-0}	=	*Length of time period between the beginning of the measurement period and the date of the interim value (V_{i-1}).*

AIRR – Example (4/6)

Date	Cash flows	Market values	Actual P&L	Period rate of return	PV of BMV	Implied P&L
31.12.2010	-100.0	100.0				
31.03.2011	0.0	105.0	5.0	5.00%	100.0	5.0
30.06.2011	0.0	111.3	6.3	6.00%	105.0	6.3
30.09.2011	20.0	86.8	-4.5	-4.00%	111.3	-4.5
31.12.2011	0.0	85.1	-1.7	-2.00%	86.8	-1.7
31.03.2012	80.9	80.9	0.0	-5.00%	85.1	-4.3
Total			5.1		488.3	0.9
Annualized total period IRR	0.74%	Cost of capital = 0.00%				
Annualized total period AIRR	0.70%					

- Here cost of capital is assumed to be 0.00% to make this example comparable to the TMWR example – in reality one may choose an explicit rate for the cost of capital.

AIRR – Comparison to the IRR (5/6)

Case	Cost of capital	Cash flows	Values	P&L	Implied P&L	AIC AIRR	AIRR
1	IRR	no	implied	actual	not actual	not AIC IRR	IRR
2	IRR	no	actual	actual	not actual	not AIC IRR	IRR
3	IRR	yes	implied	actual	not actual	not AIC IRR	IRR
4	IRR	yes	actual	actual	not actual	not AIC IRR	IRR
5	not IRR	no	implied	actual	not actual	not AIC IRR	IRR
6	not IRR	no	actual	actual	not actual	not AIC IRR	not IRR
7	not IRR	yes	implied	actual	not actual	not AIC IRR	IRR
8	not IRR	yes	actual	actual	not actual	not AIC IRR	not IRR

- Differences between AIRR and IRR are driven by the reinvestment assumption and the implied capital at risk or average invested capital (AIC).
- Case 5 and 7: because periodic rate of returns are equal for all sub-periods.
- Case 6 and 8: if CoC > IRR then IRR > AIRR and if CoC < IRR then IRR < AIRR.

AIRR – Critics (6a/6)

- AIRR is a money-weighted rate of return as it is influenced by the timing of cash flows – therefore may not useful to assess a portfolio manager without discretion over the cash in- and outflows.
- If length of measurement sub-period does not equal the averaging period (e.g. annual, quarterly) then calculation have to be adjusted.
- If the desired cash flow frequency exceeds the valuation frequency (for the actual interim values), then the implied values of the sub-period XIRRs have to be used to interpolate the missing values that AIRR requires.
- Implicit interim profit and loss is not equal to the profit and loss observed on the balance sheet – except if cost of capital equals 0.00%.
- AIRR needs actual (or estimates of the) interim values - if not available then IRR should be used, assuming interim values equals implied IRR values.
- AIRR is an average rate of return but in comparison to the IRR based on actual interim values.

AIRR – Critics (6b/6)

- AIRR contribution or decomposition not addressed yet.
- AIRR can always be calculated and is unique – means has no issue with multiple solutions.
- AIRR is quite easy to calculate and may be used as a (first) estimation of the IRR.
- AIRR is more intuitive than the IRR as the calculation is based on actual sub-period returns and on (discounted) actual interim values (market values).
- AIRR is depending on the used rate for the cost of capital.
- AIRR is path-depending with respect to the cash flows and the actual interim values.
- AIRR is useful for both ex ante investment decision-making and ex post performance evaluation, using an appropriate cost of capital.

Modified internal rate of return (MIRR)

MIRR – Starting point (1/5)

- IRR solution might not be unique - multiple solutions.
- IRR equation might not be solvable.

MIRR – Definition (2a/5)

- The modified internal rate of return (MIRR) is a modification of the internal rate of return (IRR) which uses explicit reinvestment assumptions.
- MIRR is a money-weighted rate of return and is called a "true" MWR as it is a precise method for calculating a MWR and no approximation of the MWR.
- The MIRR is the discount rate that makes the investments (cash inflows) equal to the future value of the cash flows from the investment (cash outflows).
- The MIRR is the average annual rate of return that will be earned if the external cash flows are financed or reinvested using explicit reinvestment assumptions.
- In contrast to the IRR, the MIRR uses explicit reinvestment assumptions for the cash inflows and the cash outflows.

MIRR – Definition (2b/5)

- The explicit reinvestment assumption of the MIRR methodology with respect to the interim external cash flows is that:
  1) Cash inflows are financed at an interest rate (so called finance rate) – that does not have to be identical to the IRR.
  2) Cash outflows are reinvested at an interest rate (so called reinvestment rate) – that does not have to be identical to the IRR.
- By using explicit reinvestment assumptions the MIRR resolves two issues with the IRR methodology:
  1) The (unrealistic) implicit reinvestment assumptions.
  2) The problem of having multiple solutions.

MIRR – Formula (3a/5)

$$MIRR = \left(\frac{Future\ value\ of\ all\ cash\ outflows}{Present\ value\ of\ all\ cash\ inflows} \right)^{\left(\frac{1}{Y_T}\right)} - 1$$

$$MIRR = \left(\frac{EMV + \sum_{k=1}^{K} \left(Cof_k \times (1 + ri_{t,T})^{Y_T - t} \right)}{BMV + \sum_{l=1}^{L} \frac{Cif_l}{(1 + rf_{0,t})^{Y_{t-0}}}} \right)^{\left(\frac{1}{Y_T}\right)} - 1$$

- Here consistent finance and reinvestment rates throughout the whole measurement period are assumed. The formula can be adjusted to reflect changing finance and reinvestment rates and to add other costs or revenues like taxes.

MIRR – Formula (3b/5)

Cof_k = *Cash outflow k.*
Cif_l = *Cash inflow l.*
$ri_{t,T}$ = *Reinvestment rate for the time period t to T.*
$rf_{0,t}$ = *Finance rate for the time period 0 to t.*
Y_T = *Length of measurement period (to be measured in years – 365).*
Y_{T-t} = *Length of time period between the date of the cash outflow and the end of the measurement period (to be measured in years – 365).*
Y_{t-0} = *Length of time period between the beginning of the measurement period and the date of the cash inflow (to be measured in years – 365).*

MIRR – Example (4/5)

Portfolio data	31.12.2010	31.03.2011	30.06.2011	30.09.2011	31.12.2011	31.03.2012
Assets	100.0	105.0	111.3	86.8	85.1	80.9
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TWRs		5.00%	6.00%	-4.00%	-2.00%	-5.00%
Annualized total period IRR	0.74%	Finance and reinvestment rate = 0.00%				
Annualized total period MIRR	0.68%					

- Here finance and reinvestment rate are assumed to be 0.00% to make this example comparable to the TMWR and AIRR example - in reality one may choose more realistic finance and reinvestment rates.

MIRR – Critics (5a/5)

- MIRR is a money-weighted rate of return as it is influenced by the timing of cash flows – therefore may not useful to assess a portfolio manager without discretion over the cash in- and outflows.
- MIRR is an average rate of return but in comparison to the IRR based on realistic reinvestment assumptions.
- MIRR is an average rate of return not based on the actual interim values but on the actual cash flows.
- MIRR contribution or decomposition not addressed yet.
- MIRR can always be calculated and is unique – means has no issue with multiple solutions.
- MIRR is quite easy to calculate.
- MIRR is more realistic than the IRR as the calculation is based on actual reinvestment assumptions.

MIRR – Critics (5b/5)

- MIRR is depending on the used rates for the finance rate and the reinvestment rate.
- MIRR is path-depending with respect to the cash flows.
- MIRR is useful for both ex ante investment decision-making and ex post performance evaluation, using an appropriate cost of capital.
- MIRR can be calculated using different reinvestment assumptions – e.g. risk free rate, benchmark return, target return, etc. – where in addition the assumptions do not have to be static but can be dynamic throughout the measurement period (=> dynamic reinvestment assumptions would make the MIRR more "realistic").

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